semiGroup(P) **Θ:** [

$P(P(Q,R),S){=}P(Q,P(R,S))$]
-- : **associative law**

leftMonoid(P,Q) **Θ:** [

semiGroup(P),
$P(Q,R){=}R$]
-- : **left monoid**

rightMonoid(P,Q) **Θ:** [

semiGroup(P),
$P(R,Q){=}R$]
-- : **right monoid**

monoid(P,Q) **Θ:** [

leftMonoid(P,Q),
$P(R,Q){=}R$]
-- : **bilateral monoid**

commMonoid(P,Q) **Θ:** [

leftMonoid(P,Q),
$P(R,S){=}P(S,R)$]
-- : **commutative monoid**

leftDistributes(P,Q) **Θ:** [

$P(R,Q(S,T)){=}Q(P(R,S),P(R,T))$]
-- : **left distributive law**

rightDistributes(P,Q) **Θ:** [

$P(Q(R,S),T){=}Q(P(R,T),P(S,T))$]
-- : **right distributive law**

semiGroup$(\triangle)$
leftMonoid$(\circ,\iota)$
commMonoid$(\cap,\mathbb{1})$
$(P\triangle Q)\circ R\subseteq Q\circ R\cup P\circ R$
$[P\subseteq Q]\Rightarrow P\circ R\subseteq Q\circ R$
$P\cap(Q\triangle R)\triangle P\cap Q{=}P\cap R$
$P^{\smile\smile}{=}P$
$(P\cap Q)^{\smile}{=}Q^{\smile}\cap P^{\smile}$
$(P\circ Q)^{\smile}{=}Q^{\smile}\circ P^{\smile}$
$[P\circ Q\cap R{=}\emptyset]\Rightarrow P^{\smile}\circ R\cap Q{=}\emptyset$
$\mathrm{frk}(P,Q){=}P\circ\mathrm{frk}(\iota,\mathbb{1})\cap Q\circ\mathrm{frk}(\mathbb{1},\iota)$
$\mathrm{frk}(P,Q)\circ\mathrm{frk}^{\smile}(R,S){=}P\circ R^{\smile}\cap Q\circ S^{\smile}$
$\mathrm{frk}(\mathrm{left},\mathrm{right})\cap\overline{\iota}{=}\emptyset$