semiGroup(P) **Θ:** [
$P(P(Q,R),S)=P(Q,P(R,S))$]
-- : associative law

leftMonoid(P,Q) **Θ:** [
semiGroup(P),
$P(Q,R)=R$]
-- : left monoid

rightMonoid(P,Q) **Θ:** [
semiGroup(P),
$P(R,Q)=R$]
-- : right monoid

monoid(P,Q) **Θ:** [
leftMonoid(P,Q),
$P(R,Q)=R$]
-- : bilateral monoid

commMonoid(P,Q) **Θ:** [
leftMonoid(P,Q),
$P(R,S)=P(S,R)$]
-- : commutative monoid

leftDistributes(P,Q) **Θ:** [
$P(R,Q(S,T))=Q(P(R,S),P(R,T))$]
-- : left distributive law

rightDistributes(P,Q) **Θ:** [
$P(Q(R,S),T)=Q(P(R,T),P(S,T))$]
-- : right distributive law

-- Boolean axioms (Huntington, 1933, later improved by Robbin):

$$P\cup Q=Q\cup P$$

$$\mathsf{semiGroup}(\cup)$$

$$\overline{\overline{P\cup Q}\cup\overline{P\cup\overline{Q}}}=P$$

-- associative law, and unit element, for map composition:

$$\mathsf{rightMonoid}(\circ,\boldsymbol{\iota})$$

-- distributivity of composition over union:

$$\mathsf{rightDistributes}(\circ,\cup)$$

-- convolutory laws:

$$P^{\smile\smile}=P$$

$$(P\cup Q)^{\smile}=P^{\smile}\cup Q^{\smile}$$

$$(P\circ Q)^{\smile}=Q^{\smile}\circ P^{\smile}$$

-- Schröder's cycle law:

$$P^{\smile}\circ\overline{P\circ Q}\cup\overline{Q}=\overline{Q}$$