

-- i . txt

formula\_list(usable) -- conversion laws

- $[\forall x, \forall y \mid x \subseteq y \rightarrow x^{\smile} \subseteq y^{\smile}]$  -- (i)<sub>0</sub>
- $[\forall x, \forall y \mid (x \cup y)^{\smile} = x^{\smile} \cup y^{\smile}]$  -- (i)<sub>1</sub>  $\equiv$  BVIII
- $[\forall x \mid \bar{x}^{\smile} = \overline{x^{\smile}}]$  -- (i)<sub>3</sub>
- $[\forall x, \forall y \mid (x \cap y)^{\smile} = x^{\smile} \cap y^{\smile}]$  -- (i)<sub>2</sub>
- $[\forall x \mid x^{\smile\smile} = x]$  -- (i)<sub>4</sub>  $\equiv$  BVII
- $[\forall x, \forall y \mid (x \dagger y)^{\smile} = y^{\smile} \dagger x^{\smile}]$  -- (i)<sub>5</sub>
- $[\forall x, \forall y \mid (x \circ y)^{\smile} = y^{\smile} \circ x^{\smile}]$  -- (i)<sub>6</sub>  $\equiv$  BIX
- $\iota^{\smile} = \iota$  -- (i)<sub>7a</sub>
- $\emptyset^{\smile} = \emptyset$  -- (i)<sub>7b</sub>
- $\mathbb{1}^{\smile} = \mathbb{1}$  -- (i)<sub>7c</sub>

end\_of\_list

-- ii . txt

formula\_list(usable)

- $\mathbb{1} \circ \mathbb{1} = \mathbb{1}$  -- (ii)<sub>0</sub>  $\equiv$  (ix)<sub>7</sub>
- $\mathbb{1}^{\smile} = \mathbb{1}$  -- (ii)<sub>1</sub>
- $\emptyset^{\smile} = \emptyset$  -- (ii)<sub>2</sub>
- $\iota^{\smile} = \iota$  -- (ii)<sub>3</sub>
- $\delta^{\smile} = \delta$  -- (ii)<sub>4</sub>

end\_of\_list

-- iii . txt

formula\_list(usable)

$[\forall x, \forall y \mid \overline{xoy} = \overline{x\uparrow y}]$  -- ( iii )<sub>1</sub>

$[\forall x, \forall y \mid \overline{x\uparrow y} = \overline{xoy}]$  -- ( iii )<sub>2</sub>

$[\forall x, \forall y \mid (xoy) \smile = y \smile ox \smile]$  -- ( iii )<sub>3</sub>  $\equiv$  BIX  $\equiv$  ( i )<sub>6</sub>

$[\forall x, \forall y \mid (x\uparrow y) \smile = y \smile \uparrow x \smile]$  -- ( iii )<sub>4</sub>  $\equiv$  ( i )<sub>5</sub>

end\_of\_list

-- iv . txt

formula\_list(usable)

$[\forall x, \forall y, \forall z \mid xo(yoz) = (xoy)oz]$  -- ( iv )<sub>1</sub>  $\equiv$  BIV

$[\forall x, \forall y, \forall z \mid x\uparrow(y\uparrow z) = (x\uparrow y)\uparrow z]$  -- ( iv )<sub>2</sub>

end\_of\_list

-- v . txt

formula\_list(usable)

$[\forall x, \forall y, \forall z \mid xo(yUz) = xoyUxoz]$  -- ( v )<sub>1</sub>

$[\forall x, \forall y, \forall z \mid (xUy)oz = xozUyoz]$  -- ( v )<sub>2</sub>  $\equiv$  BV

end\_of\_list

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-- ix_a . txt
formula_list(usable)
  [∀x | x ∘ ι = x]      -- ( ix )1 ≡ BVI
end_of_list

-- ix_b . txt
formula_list(usable)
  [∀x | ι ∘ x = x]      -- ( ix )2
  [∀x | x ⊆ x ∘ 1]      -- ( ix )3
  [∀x | x ⊆ 1 ∘ x]      -- ( ix )4
  [∀x | ∅ † x ⊆ x]      -- ( ix )5
  [∀x | x † ∅ ⊆ x]      -- ( ix )6
  [∀x | 1 ∘ 1 = 1]      -- ( ix )7 ≡ ( ii )0
  [∀x | ∅ † ∅ = ∅]      -- ( ix )8
end_of_list

-- xi_a . txt
formula_list(usable)
  [∀x | x ∘ x̄ ⊆ δ]      -- ( xi )4
  [∀x | ι ⊆ x̄ † x ∘]    -- ( xi )1
end_of_list

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-- xi\_b . txt

formula\_list(usable)

$$[\forall x | \iota \subseteq \bar{x} \cup x] \quad \text{-- ( xi )}_3$$

$$[\forall x | x \cap \bar{x} \subseteq \delta] \quad \text{-- ( xi )}_2$$

end\_of\_list

-- xiii . txt

formula\_list(usable)

$$[\forall x | x \circ \emptyset = \emptyset] \quad \text{-- ( xiii )}_4$$

$$[\forall x | \emptyset \circ x = \emptyset] \quad \text{-- ( xiii )}_3$$

$$[\forall x | x \dagger \mathbb{1} = \mathbb{1}] \quad \text{-- ( xiii )}_2$$

$$[\forall x | \mathbb{1} \dagger x = \mathbb{1}] \quad \text{-- ( xiii )}_1$$

end\_of\_list

-- xiv . txt

formula\_list(usable)

$$[\forall x | \mathbb{1} \circ x = \emptyset \dagger \mathbb{1} \circ x] \quad \text{-- ( xiv )}_1$$

$$[\forall x | x \circ \mathbb{1} = x \circ \mathbb{1} \dagger \emptyset] \quad \text{-- ( xiv )}_2$$

$$[\forall x | \emptyset \dagger x = \mathbb{1} \circ (\emptyset \dagger x)] \quad \text{-- ( xiv )}_3$$

$$[\forall x | x \dagger \emptyset = (x \dagger \emptyset) \circ \mathbb{1}] \quad \text{-- ( xiv )}_4$$

end\_of\_list

-- xx . txt

formula\_list(usable)

$[\forall x \mid x \circ \mathbb{1} = \mathbb{1} \rightarrow \iota \subseteq x \circ x]$  -- ( xx )<sub>a</sub>

$[\forall x \mid x \circ \mathbb{1} = \mathbb{1} \leftrightarrow \iota \subseteq x \circ x]$  -- ( xx )

end\_of\_list

-- xxii . txt

formula\_list(usable)

$[\forall x \mid x \subseteq \iota \rightarrow x \circ x = x]$  -- ( xxii )<sub>2</sub>

$[\forall x \mid x \subseteq \iota \rightarrow x^\smile = x]$  -- ( xxii )<sub>1</sub>

end\_of\_list

-- xxv . txt

formula\_list(usable)

$[\forall x, \forall y \mid \iota \subseteq x \ \& \ x^\smile \subseteq x \ \& \ x \circ x \subseteq x \rightarrow x \circ \overline{x \circ y} = \overline{x \circ y}]$  -- ( xxv )

end\_of\_list

-- vi . txt

formula\_list(usable)

$[\forall x, \forall y, \forall z \mid x \dagger y \cap z = (x \dagger y) \cap (x \dagger z)]$  -- ( vi )<sub>1</sub>

$[\forall x, \forall y, \forall z \mid x \cap y \dagger z = (x \dagger z) \cap (y \dagger z)]$  -- ( vi )<sub>2</sub>

end\_of\_list

-- vii . txt

formula\_list(usable)

$[\forall x, \forall y, \forall z \mid x \subseteq y \rightarrow xoz \subseteq yoz]$  -- ( vii )<sub>1</sub>

$[\forall x, \forall y, \forall z \mid x \subseteq y \rightarrow zox \subseteq zoy]$  -- ( vii )<sub>2</sub>

$[\forall x, \forall y, \forall z \mid x \subseteq y \rightarrow x\uparrow z \subseteq y\uparrow z]$  -- ( vii )<sub>3</sub>

$[\forall x, \forall y, \forall z \mid x \subseteq y \rightarrow z\uparrow x \subseteq z\uparrow y]$  -- ( vii )<sub>4</sub>

$[\forall x, \forall y \mid x \subseteq y \rightarrow x^{\smile} \subseteq y^{\smile}]$  -- ( vii )<sub>5</sub>

end\_of\_list

-- 1To3monotLaws . txt

formula\_list(usable) -- ( inclusion appears in disguise )

$[\forall x, \forall y, \forall z \mid x \cup y = y \rightarrow zox \cup zoy = zoy]$

$[\forall x, \forall y, \forall z \mid x \cup y = y \rightarrow \overline{z \cup y} \cup \overline{x} = \overline{x}]$

$[\forall x, \forall y, \forall z \mid x \cup y = y \rightarrow xoz \cup yoz = yoz]$

end\_of\_list

-- 4\_5monotLaws . txt

formula\_list(usable)

$[\forall x, \forall y, \forall w, \forall v \mid (x \cap w) \circ (y \cap v) \subseteq xoy]$

$[\forall x, \forall y, \forall w, \forall v \mid (x \cap w) \circ (y \cap v) \subseteq wov]$

end\_of\_list

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-- 30booleanLaws . txt
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formula_list(usable)
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  [ $\forall x, \forall y, \forall z, \forall w \mid x \cap y \cap (z \cap w) = x \cap w \cap (z \cap y)$ ]
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end_of_list
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