

```

--
-- ----- PROOF -----
-- Length of proof is 127.Level of proof is 40 .
-- 1 [ ]  $\neg x \subseteq y \vee x \cup y = y$ 
-- 2 [ ]  $x \subseteq y \vee x \cup y \neq y$ 
-- 3 [ ]  $\neg x \circ y \subseteq \bar{z} \vee x \overset{\sim}{\circ} z \subseteq \bar{y}$ 
-- 4 [ ]  $x \circ y \subseteq \bar{z} \vee \neg x \overset{\sim}{\circ} z \subseteq \bar{y}$ 
-- 5 [ ]  $\neg c3 \circ c2 \subseteq c1 \vee \neg c1 \circ c2 \overset{\sim}{\subseteq} c3$ 
-- 6 [ ]  $x \cup y = y \cup x$ 
-- 7 [ ]  $x \cup (y \cup z) = x \cup y \cup z$ 
-- 9,8 [ copy,7,flip.1 ]  $x \cup y \cup z = x \cup (y \cup z)$ 
-- 10 [ ]  $\overline{x \cup y \cup x \cup y} = x$ 
-- 16,15 [ ]  $x \overset{\sim}{\sim} = x$ 
-- 18,17 [ ]  $(x \cup y) \overset{\sim}{\sim} = x \overset{\sim}{\sim} \cup y \overset{\sim}{\sim}$ 
-- 20,19 [ ]  $(x \circ y) \overset{\sim}{\sim} = y \overset{\sim}{\sim} \circ x \overset{\sim}{\sim}$ 
-- 21 [ ]  $x \overset{\sim}{\circ} y \cup x \circ z \cup \bar{z} = \bar{z}$ 
-- 23 [ ]  $x \circ \iota = x$ 
-- 26,25 [ ]  $\iota \circ x = x$ 
-- 27 [ ]  $c3 \circ c2 \subseteq c1 \vee c1 \circ c2 \overset{\sim}{\subseteq} c3$ 

```

```

-- 29 [ para_into,6.1.1,1.2.1,flip.1 ]  $x \cup y = x \vee \neg y \subseteq x$ 
-- 30 [ para_from,6.1.1,2.2.1 ]  $x \subseteq y \vee y \cup x \neq y$ 
-- 32 [ para_into,15.1.1,15.1.1 ]  $x = x$ 
-- 33 [ para_from,23.1.1,4.2.1 ]  $x \circ y \subseteq \bar{y} \vee \neg x \subseteq \bar{y}$ 
-- 38 [ para_from,25.1.1,3.1.1 ]  $\neg x \subseteq \bar{y} \vee \neg y \subseteq \bar{x}$ 
-- 39 [ para_into,17.1.1.1,29.1.1,flip.1 ]  $x \cup y = x \vee \neg y \subseteq x$ 
-- 40 [ para_into,17.1.1.1,1.2.1,flip.1 ]  $x \cup y = y \vee \neg x \subseteq y$ 
-- 41 [ para_into,19.1.1.1,25.1.1,flip.1 ]  $x \circ \neg y = x$ 
-- 46 [ para_into,10.1.1.1.1,6.1.1 ]  $\overline{x \cup y \cup y \cup x} = y$ 
-- 48 [ para_into,10.1.1.1.1,1.2.1 ]  $\overline{x \cup y \cup x} = y \vee \neg y \subseteq x$ 
-- 49 [ para_into,10.1.1.2.1,29.1.1 ]  $\overline{x \cup y \cup x} = x \vee \neg y \subseteq \bar{x}$ 
-- 50 [ para_into,10.1.1.2.1,6.1.1 ]  $\overline{x \cup y \cup y \cup x} = x$ 
-- 53 [ para_into,10.1.1,29.1.1 ]  $\overline{x \cup y} = x \vee \neg \overline{x \cup y} \subseteq \overline{x \cup y}$ 
-- 54 [ para_into,10.1.1,6.1.1 ]  $\overline{x \cup y \cup x \cup y} = x$ 
-- 61 [ para_into,41.1.1.1,15.1.1,demod,16 ]  $x \circ \neg y = x$ 
-- 64,63 [ para_into,61.1.1,25.1.1 ]  $\neg y = \neg y$ 
-- 65 [ back_demod,38,demod,64,26 ]  $\neg x \subseteq \bar{y} \vee y \subseteq \bar{x}$ 
-- 66 [ para_into,33.1.1,23.1.1 ]  $x \subseteq \bar{y} \vee \neg x \subseteq \bar{y}$ 
-- 69 [ para_into,66.2.1,15.1.1 ]  $x \subseteq \bar{y} \vee \neg x \subseteq \bar{y}$ 

```

```

-- 72 [ para_into,21.1.1.1.1,63.1.1,demod,26,26 ]  $\overline{\overline{xUyUy}} = \overline{y}$ 
-- 77 [ para_into,39.1.1.1,15.1.1,demod,16 ]  $xUy \sim = x \vee \neg y \underline{\underline{C}} x \sim$ 
-- 80 [ para_from,39.1.1,30.2.1,unit_del,32 ]  $x \sim \underline{\underline{C}} y \sim \vee \neg x \underline{\underline{C}} y$ 
-- 82 [ para_into,80.1.1,15.1.1 ]  $x \underline{\underline{C}} y \sim \vee \neg x \sim \underline{\underline{C}} y$ 
-- 83 [ para_into,80.1.2,63.1.1 ]  $x \sim \underline{\underline{C}} \iota \vee \neg x \underline{\underline{C}} \iota$ 
-- 84 [ para_into,80.1.2,15.1.1 ]  $x \sim \underline{\underline{C}} y \vee \neg x \underline{\underline{C}} y \sim$ 
-- 89 [ hyper,27,3 ]  $c1oc2 \sim \underline{\underline{C}} c3 \vee c3 \sim oc1 \underline{\underline{C}} c2$ 
-- 111 [ para_into,40.1.1.2,63.1.1,demod,64 ]  $x \sim U \iota = \iota \vee \neg x \underline{\underline{C}} \iota$ 
-- 124 [ para_from,111.1.1,8.1.1.1,flip.1 ]  $x \sim U (\iota U y) = \iota U y \vee \neg x \underline{\underline{C}} \iota$ 
-- 160 [ para_into,48.1.1,6.1.1 ]  $\overline{\overline{xUyUy}} = x \vee \neg \overline{\overline{x}} \underline{\underline{C}} y$ 
-- 172 [ hyper,72,2 ]  $\overline{\overline{xUy}} \underline{\underline{C}} \overline{y}$ 
-- 177 [ para_into,72.1.1.1.1,77.1.1 ]  $\overline{\overline{xUy}} \sim = \overline{y} \sim \vee \neg y \underline{\underline{C}} \overline{x} \sim$ 
-- 184,183 [ para_into,72.1.1.1.1,6.1.1 ]  $\underline{\underline{xUyUx}} = \overline{\overline{x}}$ 
-- 189,188 [ para_into,72.1.1,6.1.1 ]  $\overline{\overline{xUyUx}} = \overline{\overline{x}}$ 
-- 208 [ hyper,172,65 ]  $x \underline{\underline{C}} \overline{\overline{yUx}}$ 
-- 218 [ para_into,172.1.1.1,77.1.1 ]  $\overline{\overline{x}} \underline{\underline{C}} \overline{y} \sim \vee \neg y \underline{\underline{C}} \overline{x} \sim$ 
-- 219 [ para_into,172.1.1.1,72.1.1 ]  $\overline{\overline{x}} \underline{\underline{C}} \overline{\overline{x}}$ 
-- 227 [ hyper,219,65 ]  $\overline{\overline{x}} \underline{\underline{C}} \overline{\overline{x}}$ 
-- 229,228 [ hyper,219,49,demod,184 ]  $\overline{\overline{x}} = \overline{\overline{x}}$ 

```

-- 235 [back_demod,227,demod,229] $\bar{x} \subseteq \bar{x}$
-- 238 [hyper,235,69] $\bar{t} \subseteq \bar{t}$
-- 239 [hyper,235,65] $x \subseteq \bar{x}$
-- 241,240 [hyper,235,49] $\bar{x} \cup x \cup \bar{x} = x$
-- 252 [hyper,239,82] $x \subseteq \bar{x}$
-- 258,257 [hyper,239,49,demod,229,229,241,flip.1] $\bar{x} = x$
-- 268,267 [hyper,239,29,demod,258,258] $x \cup x = x$
-- 273 [back_demod,252,demod,258,16] $x \subseteq x$
-- 277 [back_demod,218,demod,258] $x \subseteq \bar{y} \vee \neg y \subseteq \bar{x}$
-- 282 [back_demod,208,demod,258] $x \subseteq \bar{y} \cup x$
-- 297 [back_demod,177,demod,258] $x \cup \bar{y} = \bar{y} \vee \neg y \subseteq \bar{x}$
-- 323,322 [para_into,50.1.1.1.1,10.1.1,demod,258,9,189] $\bar{x} \cup \bar{x} \cup \bar{y} = \bar{x} \cup y$
-- 324 [para_into,50.1.1.1.1,1.2.1,demod,323] $\bar{x} \cup y = y \vee \neg \bar{y} \subseteq x$
-- 330 [para_from,50.1.1,30.2.1,flip.2] $\bar{x} \cup \bar{y} \subseteq \bar{y} \cup x \vee \bar{y} \cup x \neq y$
-- 336 [para_from,257.1.1,48.1.1.1,demod,258] $x \cup \bar{y} \cup x = y \vee \neg \bar{y} \subseteq \bar{x}$
-- 337 [para_from,257.1.1,50.1.1.2.1.2,demod,258] $\bar{x} \cup y \cup \bar{y} \cup x = \bar{x}$
-- 341 [para_from,257.1.1,172.1.1.1.1] $x \cup y \subseteq \bar{y}$
-- 349 [para_from,257.1.1,46.1.1.2.1.1,demod,258] $\bar{x} \cup y \cup y \cup \bar{x} = \bar{y}$
-- 357 [para_from,267.1.1,8.1.1.1,flip.1] $x \cup (x \cup y) = x \cup y$

-- 362,361 [hyper,238,40,demod,16] $\bar{i}U\bar{i} \sim = \bar{i} \sim$
-- 366,365 [hyper,238,29,demod,362] $\bar{i} \sim = \bar{i}$
-- 381 [para_into,282.1.2.1,257.1.1] $x \subseteq y U x$
-- 391,390 [hyper,381,1] $x U (y U x) = y U x$
-- 392 [para_into,381.1.2,6.1.1] $x \subseteq x U y$
-- 395 [para_into,392.1.2,48.1.1] $\bar{x} \subseteq y \vee \neg \bar{y} \subseteq x$
-- 411 [para_into,54.1.1.1.1.1,257.1.1,demod,258] $\overline{x U \bar{y} U x U y} = \bar{x}$
-- 422 [hyper,341,69] $\overline{x U \bar{i} \sim} \subseteq \bar{i}$
-- 423 [hyper,341,48,demod,258,258] $x U \overline{\overline{y U x U x}} = y U x$
-- 439 [para_into,341.1.1.1,6.1.1] $\overline{x U y} \subseteq \bar{x}$
-- 441 [para_into,341.1.2,257.1.1] $\overline{x U \bar{y}} \subseteq y$
-- 452,451 [hyper,439,29] $\bar{x} U \overline{x U y} = \bar{x}$
-- 454,453 [hyper,439,1] $\overline{x U y U \bar{x}} = \bar{x}$
-- 455 [para_into,439.1.2,257.1.1] $\overline{\bar{x} U y} \subseteq x$
-- 460 [hyper,441,84] $\overline{\overline{x U y} \sim} \subseteq y$
-- 461 [hyper,441,83] $\overline{x U \bar{i} \sim} \subseteq \bar{i}$
-- 472,471 [hyper,441,29] $\overline{x U y U \bar{x}} = x$
-- 493 [hyper,455,29] $x U \bar{x} U y = x$
-- 541 [para_into,422.1.1.1.1,8.1.1] $\overline{\overline{x U (y U \bar{i} \sim)}} \subseteq \bar{i}$

-- 624 [para_into,357.1.1,29.1.1,flip.1] $x \cup y = x \vee \neg x \cup y \subseteq x$
-- 628 [para_into,390.1.1,29.1.1,flip.1] $x \cup y = y \vee \neg x \cup y \subseteq y$
-- 702 [para_into,471.1.1.2.1,29.1.1] $x \cup \bar{y} = x \vee \neg \bar{x} \subseteq y$
-- 807 [hyper,89,3] $c3 \supset oc1 \subseteq \bar{c2} \vee c1 \supset oc3 \subseteq \bar{c2}$
-- 844 [para_into,277.2.2,365.1.1] $\iota \subseteq \bar{x} \supset \vee \neg x \subseteq \bar{\iota}$
-- 849 [para_into,844.1.2.1,15.1.1] $\iota \subseteq \bar{x} \vee \neg x \supset \subseteq \bar{\iota}$
-- 854 [para_into,849.1.2,257.1.1] $\iota \subseteq \bar{x} \vee \neg \bar{x} \supset \subseteq \bar{\iota}$
-- 998 [hyper,541,854] $\iota \subseteq x \cup (y \cup \iota)$
-- 1022 [para_into,998.1.2.2,6.1.1] $\iota \subseteq x \cup (\iota \cup y)$
-- 1033 [hyper,1022,29,demod,9,9,391] $x \cup (y \cup \iota) = x \cup (\iota \cup y)$
-- 1241 [para_from,624.1.1,381.1.2] $x \subseteq y \vee \neg y \cup x \subseteq y$
-- 1252 [para_into,1241.2.1,6.1.1] $x \subseteq y \vee \neg x \cup y \subseteq y$
-- 1439 [para_from,628.1.1,455.1.1.1] $\bar{x} \subseteq y \vee \neg \bar{y} \cup x \subseteq x$
-- 1545 [para_into,702.1.1.2,257.1.1] $x \cup y = x \vee \neg \bar{x} \subseteq \bar{y}$
-- 1668 [para_from,124.1.1,1252.2.1,unit_del,273] $x \supset \subseteq \iota \cup y \vee \neg x \subseteq \iota$
-- 1679 [hyper,1668,461,demod,16] $\bar{x} \cup \bar{\iota} \subseteq \iota \cup y$
-- 1710 [hyper,1679,395] $\bar{\iota} \cup \bar{x} \subseteq y \cup \bar{\iota}$
-- 2749 [para_from,1545.1.1,53.2.2.1,demod,258,258,unit_del,455] $\bar{\bar{x}} \cup y = x \vee \neg x \subseteq \bar{y}$
-- 3309 [para_into,330.1.2.1,493.1.1,demod,258,258,9,258,258,452,258,unit_del,32] $\overline{\overline{x \cup (y \cup \bar{x})}} \subseteq x$

-- 3357,3356 [hyper,3309,160,demod,454,258,flip.1] $x \cup (y \cup \bar{x}) = x \cup \bar{x}$
-- 4846,4845 [para_from,411.1.1,349.1.1.2.1,demod,9,472,258,258] $\overline{x \cup y} \cup x = x \cup \bar{y}$
-- 4851 [para_from,411.1.1,337.1.1.1.1,demod,258,258,9,472,258] $x \cup \overline{x \cup y} = x \cup \bar{y}$
-- 5270 [para_from,1033.1.1,381.1.2] $x \cup \iota \subseteq y \cup (\iota \cup x)$
-- 5377 [para_into,5270.1.2,6.1.1,demod,9] $x \cup \iota \subseteq \iota \cup (x \cup y)$
-- 5465 [para_into,5377.1.2.2,1.2.1] $x \cup \iota \subseteq \iota \cup y \vee \neg x \subseteq y$
-- 5925 [hyper,5465,1710,demod,4846,3357] $\iota \cup \bar{x} \subseteq \iota \cup \bar{\iota}$
-- 5998 [para_into,5925.1.1,423.1.1] $x \cup \iota \subseteq \iota \cup \bar{\iota}$
-- 6076 [para_into,5998.1.2,6.1.1] $x \cup \iota \subseteq \bar{\iota} \cup \iota$
-- 6111 [para_into,6076.1.1,8.1.1] $x \cup (\bar{y} \cup \iota) \subseteq \bar{\iota} \cup \iota$
-- 6675 [hyper,6111,1439] $\bar{\iota} \cup \iota \subseteq x$
-- 6679 [hyper,6111,1252] $x \subseteq \bar{\iota} \cup \iota$
-- 6702,6701 [hyper,6679,324] $\bar{\iota} \cup \iota \cup x = x$
-- 6735 [hyper,6675,336,demod,6702] $x \cup \bar{x} = \bar{\iota} \cup \iota$
-- 6746 [copy,6735,flip.1] $\bar{\iota} \cup \iota = x \cup \bar{x}$
-- 7131 [para_from,6735.1.1,17.1.1.1,demod,18,366,64,flip.1] $x \sim \cup \bar{x} \sim = \bar{\iota} \cup \iota$
-- 7166 [para_from,6746.1.1,6679.1.2] $x \subseteq y \cup \bar{y}$
-- 7287 [para_into,7166.1.2.2,257.1.1] $x \subseteq \bar{y} \cup y$
-- 7313,7312 [hyper,7287,702] $x \cup \bar{y} \cup y = x$

```

-- 8676 [ para_from,7131.1.1,4851.1.1.2.1,demod,7313,flip.1 ]  $x \cup \overline{x} = x$ 
-- 8679 [ hyper,8676,30 ]  $\overline{\overline{x}} \subseteq x$ 
-- 8681,8680 [ para_into,8676.1.1.1,15.1.1,demod,16 ]  $x \cup \overline{\overline{x}} = x$ 
-- 8689 [ para_from,8676.1.1,460.1.1.1.1 ]  $\overline{\overline{x}} \subseteq x$ 
-- 8722,8721 [ hyper,8679,297,demod,258,8681,258,flip.1 ]  $\overline{\overline{\overline{x}}} = x$ 
-- 8754 [ hyper,8689,2749,demod,8722,268,flip.1 ]  $\overline{\overline{\overline{x}}} = x$ 
-- 8784,8783 [ para_into,8754.1.1.1.1,15.1.1 ]  $\overline{\overline{x}} = x$ 
-- 8899 [ para_from,8783.1.1,84.2.2 ]  $x \subseteq \overline{y} \vee \neg x \subseteq \overline{\overline{y}}$ 
-- 8970 [ hyper,8899,807,demod,20,16,factor_simp ]  $c3 \cup c1 \subseteq \overline{c2}$ 
-- 9002 [ hyper,8970,80,demod,20,16,8784 ]  $c1 \cup c3 \subseteq \overline{c2}$ 
-- 9005 [ hyper,8970,4 ]  $c3 \cup c2 \subseteq \overline{c1}$ 
-- 9076 [ hyper,9002,4 ]  $c1 \cup c2 \subseteq \overline{c3}$ 
-- 9105 [ hyper,9076,5,9005 ]  $F$ 
-- ----- end of proof -----
--
--
-- Search stopped by max_proofs option .
--
-- = = = = = end of search = = = = =

```



```
--  
-- ----- statistics -----  
-- clauses given 1570  
-- clauses generated 328021  
-- clauses kept 8316  
-- clauses forward subsumed 110424  
-- clauses back subsumed 1351  
-- Kbytes malloced 4470  
--  
-- ----- times ( seconds ) -----  
-- user CPU time 29.84 ( 0 hr,0 min,29 sec )  
-- system CPU time 0.0 ( 0 hr,0 min,0 sec )  
-- wall - clock time 30 ( 0 hr,0 min,30 sec )
```