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-- ----- PROOF -----
-- Length of proof is 166 . Level of proof is 40 .
-- 1 [ ] c2≠c1
-- 2 [ ] xUy=yUx
-- 3 [ ] xU(yUz)=xUyUz
-- 5 , 4 [ copy , 3 , flip . 1 ] xUyUz=xU(yUz)
-- 6 [ ]  $\overline{xUyUxUy}=x$ 
-- 8 [ ]  $\delta=\bar{i}$ 
-- 10 , 9 [ copy , 8 , flip . 1 ]  $\bar{i}=\delta$ 
-- 11 [ ]  $\mathbb{1}=\iota\cup\delta$ 
-- 13 , 12 [ copy , 11 , flip . 1 ]  $\iota\cup\delta=\mathbb{1}$ 
-- 14 [ ]  $\emptyset=\bar{\mathbb{1}}$ 
-- 15 [ copy , 14 , flip . 1 ]  $\bar{\mathbb{1}}=\emptyset$ 
-- 17 [ ]  $\mathbb{1}=\emptyset$ 
-- 19 , 18 [ copy , 17 , flip . 1 ]  $\emptyset=\mathbb{1}$ 
-- 21 , 20 [ back_demod , 15 , demod , 19 ]  $\bar{\mathbb{1}}=\mathbb{1}$ 
-- 25 [ para_into , 4 . 1 . 1 . 1 , 2 . 1 . 1 , demod , 5 ] xU(yUz)=yU(xUz)
-- 26 [ para_into , 4 . 1 . 1 , 2 . 1 . 1 ] xU(yUz)=yU(zUx)

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-- 27 [ copy , 26 , flip . 1 ]  $x \cup (y \cup z) = z \cup (x \cup y)$ 
-- 29 , 28 [ para_into , 12 . 1 . 1 , 2 . 1 . 1 ]  $\delta U \iota = \mathbb{1}$ 
-- 31 , 30 [ para_from , 12 . 1 . 1 , 4 . 1 . 1 . 1 ]  $\mathbb{1} \cup x = \iota \cup (\delta U x)$ 
-- 32 [ para_into , 6 . 1 . 1 . 1 . 1 . 1 , 20 . 1 . 1 , demod , 31 , 21 , 31 ]  $\overline{\iota \cup (\delta U x)} \cup \overline{\iota \cup (\delta U \bar{x})} = \mathbb{1}$ 
-- 34 [ para_into , 6 . 1 . 1 . 1 . 1 . 1 , 9 . 1 . 1 , demod , 10 ]  $\delta U x \cup \delta U \bar{x} = \iota$ 
-- 36 [ para_into , 6 . 1 . 1 . 1 . 1 , 6 . 1 . 1 ]  $\overline{\overline{\overline{\bar{x} \cup \bar{x} \cup y \cup \bar{x} \cup \bar{y}}}} = \bar{x} \cup y$ 
-- 38 [ para_into , 6 . 1 . 1 . 1 . 1 , 2 . 1 . 1 ]  $\overline{\overline{\overline{x \cup \bar{y} \cup \bar{y} \cup \bar{x}}}} = y$ 
-- 41 [ para_into , 6 . 1 . 1 . 2 . 1 . 2 , 20 . 1 . 1 ]  $\overline{\overline{\overline{\bar{x} \cup \mathbb{1} \cup \bar{x} \cup \mathbb{1}}}} = x$ 
-- 43 [ para_into , 6 . 1 . 1 . 2 . 1 . 2 , 9 . 1 . 1 ]  $\overline{\overline{\overline{\bar{x} \cup \iota \cup \bar{x} \cup \delta}}}} = x$ 
-- 45 [ para_into , 6 . 1 . 1 . 2 . 1 , 6 . 1 . 1 ]  $\overline{\overline{\overline{\bar{x} \cup y \cup (\bar{x} \cup \bar{y}) \cup \bar{x}}}} = \bar{x} \cup y$ 
-- 47 [ para_into , 6 . 1 . 1 . 2 . 1 , 2 . 1 . 1 ]  $\overline{\overline{\overline{\bar{x} \cup y \cup \bar{y} \cup \bar{x}}}} = x$ 
-- 50 [ para_into , 6 . 1 . 1 , 2 . 1 . 1 ]  $\overline{\overline{\overline{\bar{x} \cup \bar{y} \cup \bar{x} \cup y}}}} = x$ 
-- 52 [ para_from , 6 . 1 . 1 , 4 . 1 . 1 . 1 , flip . 1 ]  $\overline{\overline{\overline{\bar{x} \cup y \cup (\bar{x} \cup \bar{y} \cup z)}}}} = x \cup z$ 
-- 54 [ para_into , 30 . 1 . 1 , 2 . 1 . 1 ]  $x \cup \mathbb{1} = \iota \cup (\delta U x)$ 
-- 55 [ copy , 54 , flip . 1 ]  $\iota \cup (\delta U x) = x \cup \mathbb{1}$ 
-- 64 [ para_into , 55 . 1 . 1 . 2 , 2 . 1 . 1 ]  $\iota \cup (x \cup \delta) = x \cup \mathbb{1}$ 
-- 65 [ para_into , 55 . 1 . 1 , 2 . 1 . 1 , demod , 5 ]  $\delta U (x \cup \iota) = x \cup \mathbb{1}$ 
-- 66 [ copy , 64 , flip . 1 ]  $x \cup \mathbb{1} = \iota \cup (x \cup \delta)$ 
-- 67 [ copy , 65 , flip . 1 ]  $x \cup \mathbb{1} = \delta U (x \cup \iota)$ 

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-- 68 [ para_into , 25 . 1 . 1 . 2 , 55 . 1 . 1 ]  $x \cup (y \cup \mathbb{1}) = \iota \cup (x \cup (\delta \cup y))$ 
-- 70 [ para_into , 25 . 1 . 1 . 2 , 25 . 1 . 1 ]  $x \cup (y \cup (z \cup u)) = z \cup (x \cup (y \cup u))$ 
-- 73 [ para_into , 25 . 1 . 1 . 2 , 4 . 1 . 1 , demod , 5 ]  $x \cup (y \cup (z \cup u)) = y \cup (z \cup (x \cup u))$ 
-- 74 [ para_into , 25 . 1 . 1 , 55 . 1 . 1 ]  $x \cup \mathbb{1} = \delta \cup (\iota \cup x)$ 
-- 75 [ para_into , 25 . 1 . 1 , 2 . 1 . 1 , demod , 5 ]  $x \cup (y \cup z) = x \cup (z \cup y)$ 
-- 76 [ copy , 68 , flip . 1 ]  $\iota \cup (x \cup (\delta \cup y)) = x \cup (y \cup \mathbb{1})$ 
-- 78 [ copy , 74 , flip . 1 ]  $\delta \cup (\iota \cup x) = x \cup \mathbb{1}$ 
-- 79 [ para_from , 25 . 1 . 1 , 6 . 1 . 1 . 1 . 1 ]  $\overline{x \cup (\bar{y} \cup z)} \cup \bar{y} \cup \overline{x \cup z} = y$ 
-- 81 [ para_into , 64 . 1 . 1 . 2 , 4 . 1 . 1 , demod , 5 ]  $\iota \cup (x \cup (y \cup \delta)) = x \cup (y \cup \mathbb{1})$ 
-- 82 [ copy , 81 , flip . 1 ]  $x \cup (y \cup \mathbb{1}) = \iota \cup (x \cup (y \cup \delta))$ 
-- 91 [ para_from , 67 . 1 . 1 , 6 . 1 . 1 . 1 . 1 , demod , 21 ]  $\delta \cup (\bar{x} \cup \iota) \cup \bar{x} \cup \mathbb{1} = x$ 
-- 93 [ para_from , 67 . 1 . 1 , 25 . 1 . 1 . 2 ]  $x \cup (\delta \cup (y \cup \iota)) = y \cup (x \cup \mathbb{1})$ 
-- 94 [ copy , 93 , flip . 1 ]  $x \cup (y \cup \mathbb{1}) = y \cup (\delta \cup (x \cup \iota))$ 
-- 103 [ para_into , 26 . 1 . 1 . 2 , 25 . 1 . 1 , demod , 5 ]  $x \cup (y \cup (z \cup u)) = z \cup (y \cup (u \cup x))$ 
-- 105 [ para_into , 26 . 1 . 1 . 2 , 2 . 1 . 1 ]  $x \cup (y \cup z) = z \cup (y \cup x)$ 
-- 131 [ para_into , 27 . 1 . 1 . 2 , 78 . 1 . 1 , demod , 5 ]  $x \cup (y \cup \mathbb{1}) = \iota \cup (y \cup (x \cup \delta))$ 
-- 134 [ para_into , 27 . 1 . 1 . 2 , 55 . 1 . 1 , demod , 5 ]  $x \cup (y \cup \mathbb{1}) = \delta \cup (y \cup (x \cup \iota))$ 
-- 135 [ para_into , 27 . 1 . 1 . 2 , 25 . 1 . 1 , demod , 5 ]  $x \cup (y \cup (z \cup u)) = y \cup (u \cup (x \cup z))$ 
-- 136 [ para_into , 27 . 1 . 1 . 2 , 6 . 1 . 1 , flip . 1 ]  $\bar{x} \cup \bar{y} \cup (z \cup \bar{x} \cup y) = z \cup x$ 

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-- 142 [ copy , 135 , flip . 1 ] xU(yU(zUu))=zU(xU(uUy))
-- 143 [ para_from , 27 . 1 . 1 , 6 . 1 . 1 . 1 . 1 ] xU(yUz)UyUzUx=y
-- 150 , 149 [ para_into , 75 . 1 . 1 . 2 , 74 . 1 . 1 , demod , 31 , flip . 1 ]
--      xU(iU(dUy))=xU(dU(iUy))
-- 153 , 152 [ para_into , 75 . 1 . 1 . 2 , 66 . 1 . 1 , demod , 31 , 150 ]
--      xU(iU(yUd))=xU(dU(iUy))
-- 204 , 203 [ para_from , 32 . 1 . 1 , 6 . 1 . 1 . 2 . 1 , demod , 150 , 21 ]
--      iU(dUx)U(dU(iUx))U1=iU(dUx)
-- 230 , 229 [ para_into , 34 . 1 . 1 . 1 . 1 , 74 . 1 . 1 , demod , 13 , 21 ] dU1UdU1=i
-- 235 [ para_into , 34 . 1 . 1 . 1 . 1 , 28 . 1 . 1 , demod , 21 , 10 , 31 ] iU(dUdUd)=i
-- 243 [ para_into , 34 . 1 . 1 . 1 . 1 , 2 . 1 . 1 ] xUdUdUx=i
-- 245 [ para_into , 34 . 1 . 1 . 2 . 1 , 2 . 1 . 1 ] dUxUxUd=i
-- 247 [ para_into , 34 . 1 . 1 , 2 . 1 . 1 ] dUxUdUx=i
-- 261 [ para_into , 235 . 1 . 1 . 2 , 2 . 1 . 1 ] iU(dUdUd)=i
-- 263 [ para_into , 235 . 1 . 1 , 105 . 1 . 1 , demod , 29 ] dUdU1=i
-- 269 [ para_from , 235 . 1 . 1 , 32 . 1 . 1 . 2 . 1 , demod , 10 ] iU(dU(dUd))Ud=1
-- 286 , 285 [ para_from , 263 . 1 . 1 , 25 . 1 . 1 . 2 , flip . 1 ] dUdU(xU1)=xU
-- 324 , 323 [ para_into , 36 . 1 . 1 . 2 . 1 . 2 . 1 . 1 , 32 . 1 . 1 , demod , 150 , 21 , 21 ,
-- 204 , 150 ] iU(dUx)UiU(dUx)=iU(dUx)U(dU(iUx))
-- 393 [ para_into , 38 . 1 . 1 . 2 . 1 . 2 , 9 . 1 . 1 ] iUxUxUd=x
-- 425 [ para_into , 243 . 1 . 1 . 2 . 1 , 2 . 1 . 1 ] xUdUxUd=i
-- 459 [ para_into , 245 . 1 . 1 . 1 . 1 , 66 . 1 . 1 , demod , 21 , 31 , 324 ]
--      iU(dUd)U(dU(iUd))=i
-- 469 [ para_into , 245 . 1 . 1 , 2 . 1 . 1 ] xUdUdUx=i

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-- 514 , 513 [ para_from , 247 . 1 . 1 , 6 . 1 . 1 . 2 . 1 , demod , 10 ]  $\overline{\delta U \bar{x} U (\delta U x)} \cup \delta = \delta U \bar{x}$ 
-- 519 [ para_into , 269 . 1 . 1 . 1 . 1 , 105 . 1 . 1 , demod , 29 , 5 ]  $\delta U (\delta U \mathbb{1}) \cup \delta = \mathbb{1}$ 
-- 528 , 527 [ para_from , 269 . 1 . 1 , 245 . 1 . 1 . 2 . 1 , demod , 150 , 153 , 13 , 21 ]
--  $\overline{\delta U (\delta U (\delta U \mathbb{1}))} \cup \mathbb{1} = \iota$ 
-- 566 , 565 [ para_from , 519 . 1 . 1 , 64 . 1 . 1 . 2 , flip . 1 ]  $\overline{\delta U (\delta U \mathbb{1})} \cup \mathbb{1} = \iota \cup \mathbb{1}$ 
-- 574 [ para_from , 519 . 1 . 1 , 27 . 1 . 1 . 2 , flip . 1 ]  $\delta U (x \cup \overline{\delta U (\delta U \mathbb{1})}) = x \cup \mathbb{1}$ 
-- 592 , 591 [ para_into , 285 . 1 . 1 . 2 , 4 . 1 . 1 , demod , 5 ]  $\overline{\delta U \delta U (x \cup (y \cup \mathbb{1}))} = x \cup (y \cup \iota)$ 
-- 603 [ para_into , 425 . 1 . 1 , 2 . 1 . 1 ]  $\overline{\bar{x} \cup \delta U x \cup \delta} = \iota$ 
-- 625 [ para_into , 43 . 1 . 1 . 2 . 1 , 519 . 1 . 1 , demod , 21 ]  $\overline{\delta U (\delta U \mathbb{1})} \cup \iota \cup \mathbb{1} = \delta U (\delta U \mathbb{1})$ 
-- 629 [ para_into , 43 . 1 . 1 . 2 . 1 , 2 . 1 . 1 ]  $\overline{\bar{x} \cup \iota \cup \delta U \bar{x}} = x$ 
-- 688 , 687 [ para_from , 469 . 1 . 1 , 6 . 1 . 1 . 2 . 1 , demod , 10 ]  $\overline{\bar{x} \cup \delta U (\delta U x)} \cup \delta = \bar{x} \cup \delta$ 
-- 709 [ para_from , 527 . 1 . 1 , 41 . 1 . 1 . 2 . 1 , demod , 528 , 10 , 10 , flip . 1 ]
--  $\delta U (\delta U (\delta U \mathbb{1})) = \delta U \delta$ 
-- 765 [ para_into , 45 . 1 . 1 . 1 . 1 . 2 , 469 . 1 . 1 ]  $\overline{\overline{\bar{x} \cup \delta U (\delta U x)} \cup \iota \cup \bar{x} \cup \delta} = \overline{\bar{x} \cup \delta U (\delta U x)}$ 
-- 771 [ para_into , 45 . 1 . 1 . 1 . 1 . 2 , 247 . 1 . 1 ]  $\overline{\overline{\delta U \bar{x} U (\delta U x)} \cup \iota \cup \delta U \bar{x}} = \overline{\delta U \bar{x} U (\delta U x)}$ 
-- 820 , 819 [ para_from , 709 . 1 . 1 , 55 . 1 . 1 . 2 , demod , 5 , 5 , 31 , 150 ]
--  $\iota \cup (\delta U \delta) = \delta U (\delta U (\delta U (\iota \cup \mathbb{1})))$ 
-- 829 [ back_demod , 459 , demod , 820 ]  $\delta U (\delta U (\delta U (\iota \cup \mathbb{1}))) \cup (\delta U (\iota \cup \delta)) = \iota$ 
-- 838 , 837 [ para_from , 565 . 1 . 1 , 285 . 1 . 1 . 2 , demod , 286 , flip . 1 ]
--  $\delta U (\delta U \mathbb{1}) \cup \iota = \iota \cup \iota$ 
-- 840 , 839 [ para_from , 565 . 1 . 1 , 41 . 1 . 1 . 2 . 1 , demod , 566 ]  $\overline{\iota \cup \mathbb{1} \cup \iota \cup \mathbb{1}} = \delta U (\delta U \mathbb{1})$ 
-- 847 , 846 [ back_demod , 625 , demod , 838 ]  $\overline{\iota \cup \iota \cup \mathbb{1}} = \delta U (\delta U \mathbb{1})$ 
-- 853 [ para_from , 603 . 1 . 1 , 45 . 1 . 1 . 1 . 1 . 2 ]  $\overline{\overline{\bar{x} \cup \delta U (x \cup \delta)} \cup \iota \cup \bar{x} \cup \delta} = \overline{\bar{x} \cup \delta U (x \cup \delta)}$ 

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-- 858 , 857 [ para_from , 603 . 1 . 1 , 6 . 1 . 1 . 2 . 1 , demod , 10 ]  $\overline{\overline{\overline{\overline{xU\delta U(xU\delta)U\delta}}}} = \overline{\overline{\overline{\overline{xU\delta}}}}$ 
-- 978 , 977 [ para_from , 846 . 1 . 1 , 285 . 1 . 1 . 2 , demod , 592 , 29 , flip . 1 ]
--  $\overline{\overline{\overline{\overline{\iota U \iota U \iota}}}} = \delta U \mathbb{1}$ 
-- 984 , 983 [ para_from , 846 . 1 . 1 , 4 . 1 . 1 . 1 , demod , 5 , 5 , 31 , 150 , 31 ,
-- 150 , flip . 1 ]  $\overline{\overline{\overline{\overline{\iota U \iota U (\delta U (\iota U x))}}}} = \delta U (\delta U (\delta U (\iota U x)))$ 
-- 1093 [ para_into , 50 . 1 . 1 . 2 . 1 , 34 . 1 . 1 , demod , 10 ]  $\overline{\overline{\overline{\overline{\delta U x U \delta U \overline{\overline{\overline{\overline{x}}}}}}}} = \delta U x$ 
-- 1236 [ para_into , 52 . 1 . 1 . 1 . 1 , 846 . 1 . 1 , demod , 21 , 847 , 5 ]
--  $\overline{\overline{\overline{\overline{\delta U (\delta U \mathbb{1}) U (\delta U (\delta U \mathbb{1}) U x)}}}} = \iota U (\iota U x)$ 
-- 1240 [ para_into , 52 . 1 . 1 . 1 . 1 , 565 . 1 . 1 , demod , 21 , 566 , 5 , 5 , 31 , 150 ]
--  $\overline{\overline{\overline{\overline{\iota U \mathbb{1} U (\iota U \mathbb{1} U x)}}}} = \delta U (\delta U (\delta U (\iota U x)))$ 
-- 1561 [ para_from , 94 . 1 . 1 , 32 . 1 . 1 . 1 . 1 , demod , 21 ]  $\overline{\overline{\overline{\overline{\delta U (\delta U (\iota U \iota)) U \iota U (\delta U \mathbb{1})}}}} = \mathbb{1}$ 
-- 3020 [ para_into , 91 . 1 . 1 . 2 . 1 , 66 . 1 . 1 ]  $\overline{\overline{\overline{\overline{\delta U (\overline{\overline{\overline{\overline{xU\iota}}}})} U \iota U (\overline{\overline{\overline{\overline{xU\delta}}}})}} = x$ 
-- 3062 [ para_into , 393 . 1 . 1 . 2 . 1 , 2 . 1 . 1 ]  $\overline{\overline{\overline{\overline{\iota U \overline{\overline{\overline{\overline{x}}}}}}}} = x$ 
-- 3119 [ para_into , 629 . 1 . 1 . 1 . 1 . 1 , 20 . 1 . 1 , demod , 31 , 29 , 21 ]  $\overline{\overline{\overline{\overline{\iota U \mathbb{1} U \delta U \mathbb{1}}}}} = \mathbb{1}$ 
-- 3176 [ para_into , 3119 . 1 . 1 . 1 . 1 , 74 . 1 . 1 ]  $\overline{\overline{\overline{\overline{\delta U (\iota U \iota) U \delta U \mathbb{1}}}}} = \mathbb{1}$ 
-- 3178 [ para_into , 3119 . 1 . 1 , 2 . 1 . 1 ]  $\overline{\overline{\overline{\overline{\delta U \mathbb{1} U \iota U \mathbb{1}}}}} = \mathbb{1}$ 
-- 3188 , 3187 [ para_from , 3119 . 1 . 1 , 47 . 1 . 1 . 2 . 1 , demod , 21 ]  $\overline{\overline{\overline{\overline{\delta U \mathbb{1} U (\iota U \mathbb{1}) U \mathbb{1}}}}} = \delta U \mathbb{1}$ 
-- 3202 [ para_from , 3119 . 1 . 1 , 36 . 1 . 1 . 2 . 1 . 2 . 1 . 1 , demod , 21 , 21 ]
--  $\overline{\overline{\overline{\overline{\overline{\overline{\overline{\overline{\iota U \mathbb{1} U \iota U \mathbb{1} U (\delta U \mathbb{1}) U \mathbb{1}}}}}}}} = \overline{\overline{\overline{\overline{\iota U \mathbb{1} U (\delta U \mathbb{1})}}}}$ 
-- 3205 , 3204 [ para_from , 3119 . 1 . 1 , 6 . 1 . 1 . 2 . 1 , demod , 21 ]
--  $\overline{\overline{\overline{\overline{\iota U \mathbb{1} U (\delta U \mathbb{1}) U \mathbb{1}}}}} = \iota U \mathbb{1}$ 
-- 3233 , 3232 [ back_demod , 3202 , demod , 3205 , 840 , flip . 1 ]  $\overline{\overline{\overline{\overline{\iota U \mathbb{1} U (\delta U \mathbb{1})}}}} = \delta U (\delta U \mathbb{1})$ 
-- 3235 [ para_into , 3178 . 1 . 1 . 2 . 1 , 74 . 1 . 1 ]  $\overline{\overline{\overline{\overline{\delta U \mathbb{1} U \delta U (\iota U \iota)}}}} = \mathbb{1}$ 
-- 3246 , 3245 [ para_from , 3178 . 1 . 1 , 45 . 1 . 1 . 1 . 1 . 2 , demod , 3188 , 230 ,
-- flip . 1 ]  $\overline{\overline{\overline{\overline{\delta U \mathbb{1} U (\iota U \mathbb{1})}}}} = \iota$ 
-- 3248 , 3247 [ para_from , 3178 . 1 . 1 , 50 . 1 . 1 . 1 . 1 , demod , 21 , 3246 , 10 , 31 ,
-- 820 ]  $\overline{\overline{\overline{\overline{\delta U (\delta U (\delta U (\iota U \mathbb{1})))}}}} = \delta U \mathbb{1}$ 

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-- 3263 [ back_demod , 829 , demod , 3248 ]  $\overline{\delta U \mathbb{1} U (\delta U (\iota U \delta))} = \iota$ 
-- 3500 [ para_into , 3245 . 1 . 1 . 1 . 1 , 66 . 1 . 1 ]  $\overline{\iota U (\delta U \delta) U (\iota U \mathbb{1})} = \iota$ 
-- 3514 [ para_into , 3245 . 1 . 1 , 82 . 1 . 1 , demod , 13 ]  $\iota U (\overline{\delta U \mathbb{1} U \mathbb{1}}) = \iota$ 
-- 3522 , 3521 [ para_from , 3245 . 1 . 1 , 73 . 1 . 1 . 2 , flip . 1 ]  $\overline{\delta U \mathbb{1} U (\iota U (x U \mathbb{1}))} = x U \iota$ 
-- 3543 [ para_into , 3514 . 1 . 1 . 2 . 1 . 1 , 66 . 1 . 1 ]  $\iota U (\overline{\iota U (\delta U \delta) U \mathbb{1}}) = \iota$ 
-- 3604 [ para_from , 3500 . 1 . 1 , 27 . 1 . 1 . 2 , demod , 5 , 31 , 150 , flip . 1 ]
--       $\iota U (\delta U (\iota U (x U \overline{\iota U (\delta U \delta)}))) = x U \iota$ 
-- 3686 [ para_into , 103 . 1 . 1 . 2 . 2 , 38 . 1 . 1 ]  $x U (y U z) = \overline{u U \bar{z} U (y U (\bar{z} U \bar{u} U x))}$ 
-- 3743 [ copy , 3686 , flip . 1 ]  $\overline{x U \bar{y} U (z U (\bar{y} U \bar{x} U u))} = u U (z U y)$ 
-- 4037 [ para_from , 3543 . 1 . 1 , 70 . 1 . 1 . 2 . 2 , flip . 1 ]
--       $\iota U (x U (y U (\overline{\iota U (\delta U \delta) U \mathbb{1}}))) = x U (y U \iota)$ 
-- 4084 [ para_into , 3062 . 1 . 1 , 2 . 1 . 1 ]  $\overline{\delta U \bar{x} U \iota U \bar{x}} = x$ 
-- 4369 [ para_from , 3176 . 1 . 1 , 36 . 1 . 1 . 2 . 1 . 2 . 1 . 1 , demod , 21 , 21 ]
--       $\overline{\overline{\overline{\delta U (\iota U \iota) U \delta U (\iota U \iota) U (\delta U \mathbb{1}) U \mathbb{1}}}} = \overline{\delta U (\iota U \iota) U (\delta U \mathbb{1})}$ 
-- 4372 , 4371 [ para_from , 3176 . 1 . 1 , 6 . 1 . 1 . 2 . 1 , demod , 21 ]
--       $\overline{\delta U (\iota U \iota) U (\delta U \mathbb{1}) U \mathbb{1}} = \delta U (\iota U \iota)$ 
-- 4402 , 4401 [ back_demod , 4369 , demod , 4372 ]  $\overline{\overline{\delta U (\iota U \iota) U \delta U (\iota U \iota)}} = \overline{\delta U (\iota U \iota) U (\delta U \mathbb{1})}$ 
-- 4404 [ para_into , 3235 . 1 . 1 . 1 . 1 , 66 . 1 . 1 ]  $\overline{\iota U (\delta U \delta) U \delta U (\iota U \iota)} = \mathbb{1}$ 
-- 4709 [ para_from , 3263 . 1 . 1 , 76 . 1 . 1 . 2 , demod , 5 , 3522 , flip . 1 ]  $\overline{\delta U \iota} = \iota U \iota$ 
-- 4761 , 4760 [ para_from , 4709 . 1 . 1 , 65 . 1 . 1 . 2 , flip . 1 ]  $\overline{\delta U \mathbb{1}} = \delta U (\iota U \iota)$ 
-- 4762 [ para_from , 4709 . 1 . 1 , 91 . 1 . 1 . 1 . 1 . 2 , demod , 4761 , 4402 ]
--       $\overline{\delta U (\iota U \iota) U (\delta U \mathbb{1})} = \delta$ 
-- 4776 [ para_from , 4709 . 1 . 1 , 52 . 1 . 1 . 1 . 1 , demod , 10 ]  $\overline{\iota U \iota U (\overline{\delta U \delta U x})} = \delta U x$ 
-- 5069 , 5068 [ para_into , 4762 . 1 . 1 . 1 . 1 , 105 . 1 . 1 , demod , 13 , 3233 ]
--       $\delta U (\delta U \mathbb{1}) = \delta$ 

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-- 5143 , 5142 [ back_demod , 1236 , demod , 5069 , 5069 ]  $\overline{\delta U}(\overline{\delta U}x)=\iota U(\iota Ux)$ 
-- 5154 [ back_demod , 846 , demod , 5069 ]  $\overline{\iota U \iota U 1}=\delta$ 
-- 5175 , 5174 [ back_demod , 574 , demod , 5069 ]  $\delta U(xU\overline{\delta})=xU1$ 
-- 5185 , 5184 [ back_demod , 519 , demod , 5069 ]  $\overline{\delta U}\delta=1$ 
-- 5193 , 5192 [ back_demod , 4776 , demod , 5185 , 21 , 31 , 150 , 984 ]
--  $\delta U(\delta U(\delta U(\iota Ux)))=\delta Ux$ 
-- 5196 [ back_demod , 1240 , demod , 5193 ]  $\overline{\iota U 1 U}(\overline{\iota U 1 U}x)=\delta Ux$ 
-- 5238 [ para_into , 5068 . 1 . 1 , 131 . 1 . 1 ]  $\iota U(\delta U(\delta U\delta))=\delta$ 
-- 5261 , 5260 [ para_from , 5154 . 1 . 1 , 285 . 1 . 1 . 2 , demod , 978 ]  $\overline{\delta U \delta U \delta}=\delta U 1$ 
-- 5271 , 5270 [ back_demod , 261 , demod , 5261 ]  $\overline{\iota U(\delta U 1)}=\iota$ 
-- 5272 [ back_demod , 1561 , demod , 5271 , 10 ]  $\delta U(\overline{\delta U(\iota U \iota)})U\delta=1$ 
-- 5277 , 5276 [ para_into , 5270 . 1 . 1 , 134 . 1 . 1 ]  $\delta U(\delta U(\iota U \iota))=\iota$ 
-- 5281 , 5280 [ back_demod , 5272 , demod , 5277 , 10 ]  $\delta U\delta=1$ 
-- 5287 , 5286 [ back_demod , 5238 , demod , 5281 , 5271 ]  $\iota=\delta$ 
-- 5296 , 5295 [ back_demod , 4404 , demod , 5287 , 5281 , 5287 , 5287 , 5281 , 230 , 5287 ,
-- flip . 1 ]  $1=\delta$ 
-- 5302 , 5301 [ back_demod , 4037 , demod , 5287 , 5287 , 5281 , 5296 , 5281 , 5296 , 5296 ,
-- 5185 , 5296 , 5287 ]  $\delta U(xU(yU\delta))=xU(yU\delta)$ 
-- 5308 , 5307 [ back_demod , 3604 , demod , 5287 , 5287 , 5287 , 5281 , 5296 , 5281 , 5296 ,
-- 5302 , 5287 ]  $\delta U(xU\delta)=xU\delta$ 
-- 5331 [ back_demod , 5196 , demod , 5287 , 5296 , 5281 , 5296 , 5287 , 5296 , 5281 , 5296 ,
-- 5143 , 5287 , 5287 ]  $\delta U(\delta Ux)=\delta Ux$ 
-- 5376 , 5375 [ back_demod , 4084 , demod , 5287 ]  $\overline{\delta U \bar{x} U} \overline{\delta U \bar{x}}=x$ 
-- 5382 , 5381 [ back_demod , 3020 , demod , 5287 , 5308 , 5287 , 5308 ]  $\overline{\bar{x} U \delta U \bar{x} U \delta}=x$ 

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-- 5395 [ back_demod , 853 , demod , 5287 , 858 , 5382 , flip . 1 ]  $\overline{xU\delta}U(xU\delta)=x$ 
-- 5397 [ back_demod , 771 , demod , 5287 , 514 , 5376 , flip . 1 ]  $\overline{\delta U\bar{x}}U(\delta Ux)=x$ 
-- 5401 [ back_demod , 765 , demod , 5287 , 688 , 5382 , flip . 1 ]  $\overline{xU\delta}U(\delta Ux)=x$ 
-- 5415 [ back_demod , 425 , demod , 5287 ]  $xU\delta U\bar{x}U\bar{\delta}=\delta$ 
-- 5426 , 5425 [ back_demod , 9 , demod , 5287 ]  $\bar{\delta}=\delta$ 
-- 5432 , 5431 [ para_into , 5395 . 1 . 1 , 105 . 1 . 1 ]  $\delta U(xU\overline{xU\delta})=x$ 
-- 5456 [ para_from , 5395 . 1 . 1 , 25 . 1 . 1 . 2 , flip . 1 ]  $\overline{xU\delta}U(yU(xU\delta))=yUx$ 
-- 5470 [ para_into , 5397 . 1 . 1 , 27 . 1 . 1 ]  $xU(\overline{\delta U\bar{x}U\delta})=x$ 
-- 5516 [ para_into , 5401 . 1 . 1 . 2 , 5307 . 1 . 1 ]  $xU\delta U\delta U(xU\delta)=xU\delta$ 
-- 5532 , 5531 [ para_from , 5431 . 1 . 1 , 5331 . 1 . 1 . 2 , demod , 5432 ]  $\delta Ux=x$ 
-- 5536 , 5535 [ para_from , 5431 . 1 . 1 , 25 . 1 . 1 , demod , 5532 , flip . 1 ]  $xU\overline{xU\delta}=x$ 
-- 5540 , 5539 [ para_from , 5431 . 1 . 1 , 2 . 1 . 1 , demod , 5536 , flip . 1 ]  $xU\delta=x$ 
-- 5554 , 5553 [ back_demod , 5470 , demod , 5532 , 5540 ]  $xU\bar{x}=x$ 
-- 5568 , 5567 [ back_demod , 1093 , demod , 5532 , 5532 , 5554 , 5540 , 5532 ]  $\bar{\bar{x}}=x$ 
-- 5572 , 5571 [ back_demod , 5516 , demod , 5540 , 5540 , 5568 , 5540 , 5540 ]  $xUx=x$ 
-- 5576 , 5575 [ back_demod , 5456 , demod , 5540 , 5568 , 5540 ]  $xU(yUx)=yUx$ 
-- 5577 [ back_demod , 5415 , demod , 5540 , 5540 , 5568 ]  $\bar{x}Ux=\delta$ 
-- 5590 , 5589 [ para_from , 5567 . 1 . 1 , 50 . 1 . 1 . 1 . 1 . 1 , demod , 5568 ]  $\overline{xUy}U\overline{xUy}=\bar{x}$ 
-- 5596 , 5595 [ para_from , 5571 . 1 . 1 , 136 . 1 . 1 . 2 , demod , 5590 , 5568 , flip . 1 ]
--  $\overline{\bar{x}Uy}Ux=x$ 

```

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-- 5643 , 5642 [ para_from , 5577 . 1 . 1 , 136 . 1 . 1 . 2 , demod , 5540 , 5568 , 5 ]
--           $\overline{xUy} = \overline{xU(yUx)}$ 
-- 5649 , 5648 [ para_from , 5577 . 1 . 1 , 143 . 1 . 1 . 1 . 1 , demod , 5426 , 5643 , 5 ,
-- 5596 , 5532 ]           $\overline{xU(yUx)} = x$ 
-- 5653 , 5652 [ para_from , 5577 . 1 . 1 , 143 . 1 . 1 . 1 . 1 . 2 , demod , 5540 , 5643 , 5 ,
-- 5576 , 5649 ]           $\overline{xUy} = y$ 
-- 5655 , 5654 [ para_from , 5577 . 1 . 1 , 79 . 1 . 1 . 1 . 1 . 2 , demod , 5540 , 5653 , 5568 ,
-- 5653 ]           $xUy = y$ 
-- 5658 , 5657 [ para_from , 5577 . 1 . 1 , 142 . 1 . 1 . 2 , demod , 5655 , 5655 , 5655 , 5655
-- 5655 , flip . 1 ]           $\overline{x} = \delta$ 
-- 5660 [ back_demod , 3743 , demod , 5658 , 5655 , 5658 , 5658 , 5658 , 5572 , 5658 , 5655 ,
-- 5655 , 5655 , 5655 , 5655 ]           $x = y$ 
-- 5661 [ binary , 5660 . 1 , 1 . 1 ] F
-- _____ end of proof _____

```

```

--
--

```

Search stopped by max_proofs option .

```

--

```

==== end of search =====

```

--

```

_____ statistics _____

```

-- clauses given 162
-- clauses generated 11321
-- clauses kept 3840
-- clauses forward subsumed 10625

```

```
-- clauses back subsumed 379
-- Kbytes malloced 4183
--
-- _____ times ( seconds ) _____
-- user CPU time 1 . 82 ( 0 hr , 0 min , 1 sec )
-- system CPU time 0 . 0 ( 0 hr , 0 min , 0 sec )
-- wall - clock time 2 ( 0 hr , 0 min , 2 sec )
```