semiGroup(P) **Θ:** [
$P(P(Q,R),S)=P(Q,P(R,S))$]
-- : associative law

leftMonoid(P,Q) **Θ:** [
semiGroup(P),
$P(Q,R)=R$]
-- : left monoid

rightMonoid(P,Q) **Θ:** [
semiGroup(P),
$P(R,Q)=R$]
-- : right monoid

monoid(P,Q) **Θ:** [
leftMonoid(P,Q),
$P(R,Q)=R$]
-- : bilateral monoid

commMonoid(P,Q) **Θ:** [
leftMonoid(P,Q),
$P(R,S)=P(S,R)$]
-- : commutative monoid

leftDistributes(P,Q) **Θ:** [
$P(R,Q(S,T))=Q(P(R,S),P(R,T))$]
-- : left distributive law

rightDistributes(P,Q) **Θ:** [
$P(Q(R,S),T)=Q(P(R,T),P(S,T))$]
-- : right distributive law

semiGroup$(\triangle)$
leftMonoid$(\circ,\boldsymbol{\iota})$
commMonoid$(\cap,1)$
rightDistributes$(\circ,\cup)$
$P\cap(Q\triangle R)\triangle P\cap Q=P\cap R$
$P^{\smile\smile}=P$
$(P\cap Q)^{\smile}=Q^{\smile}\cap P^{\smile}$
$(P\circ Q)^{\smile}=Q^{\smile}\circ P^{\smile}$
$P^{\smile}\circ(\_-P\circ Q)\cap Q=0$